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Why the poor prefer to stay poor

by

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Abstract: From theory we know that the time-preference rate and the intertemporal elasticity of substitution have independent effects on individual savings. We argue that the latter should be more emphasized as an explanation for low consumption traps among the poor in developing countries. As the permanently sustainable consumption converges towards a subsistence level, we argue that the absolute value of the elasticity of the marginal utility of consumption is likely to converge towards infinity, and consequently, the intertemporal elasticity of substitution converges towards zero. As a result, even far-sighted households may prefer to stay poor.

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1. Introduction

This note is motivated by the observation that informal borrowers in developing countries are regularly willing to pay real annual interest rates in the range of 25% - 100%. The development literature has for the most part focused on the supply side of the market in explaining these high interest rates, see Basu (1997) for an introduction. The present note will focus on the demand side, and discuss why borrowers are repeatedly willing to pay interest rates in this range. That is, we do not discuss why borrowers at times are willing to pay high interest rates for short-term loans, but rather why they repeatedly do so, rather than increasing their savings. Our argument is based on standard microeconomic theory for intertemporal allocation of consumption. But, we argue that the focus should be shifted from myopic preferences to myopic behavior that we explain by a low intertemporal elasticity of substitution, which in turn is determined by a near subsistence permanent income level.

Suppose that borrowers inherit initial loans that allow only subsistence consumption after the interest is paid, as described in Bhaduri's (1973) model of agrarian stagnation. The initial loans might be for emergencies, such as medical treatment, marriage or consumption in a year of a bad harvest. The stagnation part of Bhaduri's model has been heavily criticized, see Srinivasan (1978) and Basu (1997). However, our focus is on the consumption trap part of the model. Although Basu (1997) agrees that the low-consumption trap in Bhaduri's model is plausible, he argues that the poverty trap is not stable. Infinitesimal savings will in the long run enable the poor to leave the poverty trap. Basu concludes that the poor in Bhaduri's model are either extremely myopic, or the model is inadequately specified.

The present paper addresses this question. We argue that even though the poor are not myopic, their disutility of savings may explain a very low, and even zero at the limit, rate of consumption growth. That is, we argue that the intertemporal elasticity of substitution may converge towards zero as the permanently sustainable consumption converges towards a lower bound, which we interpret as the subsistence level of consumption.

To focus on intertemporal *preferences*, we keep the intertemporal budget as simple as possible. That is, we assume a fixed exogenous annual income. Furthermore, we only study households with time-preference rates below the interest rate, which means that they are net-savers. Finally, we assume that the initial negative wealth allows a permanent consumption at or above the subsistence level. This assumption might be interpreted as a result of lenders' decisions prior to period 1. That is, the lenders do not allow borrowers to have loans that they cannot manage. As a consequence, we may imagine that in case of an emergency, borrowers will receive a gift rather than an additional loan.

This leads us to other explanations for Bhaduri's debt trap that we will *not* cover in the present paper. The first alternative is a debt-trap model, where we might imagine that the lenders allow non-sustainable loans, and then add any unpaid interest to the principal. In such a model, any affordable savings may never cover the annual interest payments. Within a standard model of intertemporal allocation, this means that the permanent income is negative, but lenders allow positive consumption as long as the borrower accepts further increases in the debt. This model is likely to be relevant for a segment of the poor, but the present paper will not cover this kind of debt-trap.

The second alternative model, which we will not cover, is a model of precautionary spending. As Deaton (1992) notes, on page 29, whenever the per-period marginal utility function is concave (which might be realistic for low consumption levels), then a mean-preserving increase in risk will imply an increase in current consumption. That is, if the poor expect their future income to become more uncertain, then current consumption becomes relatively more attractive at the margin. The present paper assumes a fixed annual income, and will thus not cover any risk-based explanation for the poverty trap.

Before we present our model, we will remind the reader that we assume that all households are far-sighted, and we thus rule out an explanation for the poverty trap that is based on myopic time-preferences. Let us now turn to the model, where limited savings among the poor are explained by an intertemporal elasticity of substitution that converges towards zero as the permanently sustainable consumption converges towards a lower bound that we interpret as a subsistence level. In more popular terms, the poverty trap is explained by a relatively high marginal disutility of savings for the poor.

2. Model

We apply a standard model of intertemporal allocation of consumption, as described in detail by Deaton (1992). A consumer allocates consumption c_t over the years denoted by t to maximize his utility

$$U = \sum_{t=1}^{\infty} \frac{u(c_t)}{(1+\delta)^t}, \quad (1)$$

where $u' > 0$ and $u'' < 0$, subject to the intertemporal budget,

$$\sum_{t=1}^{\infty} \frac{c_t}{(1+r)^t} = A_0 + \sum_{t=1}^{\infty} \frac{y}{(1+r)^t}, \quad (2)$$

where y is a fixed annual income and A_0 is an initial (negative) wealth. The infinite horizon simplifies the presentation. We may imagine a dynasty of decision-makers where the present one cares as much for the next generations as for himself, except for the discounting that is fully captured by the time-preference rate. From the first-order conditions we have, as an approximation,

$$g = \sigma(r - \delta), \quad (3)$$

where g is the annual growth rate of consumption, r is the annual interest rate, δ is the time-preference rate, and σ is the intertemporal elasticity of substitution, that is $\sigma(c_t) = \frac{-u'(c_t)}{u''(c_t)c_t}$. To simplify notation we also need the inverse of this expression, $R = \sigma(c_t)^{-1} = \frac{-u''(c_t)c_t}{u'(c_t)}$, which is often referred to as the Arrow-Pratt coefficient of relative risk-aversion. The R is also the absolute value of the elasticity of the marginal utility of consumption. Since we do not allow for risk in the present paper, the appropriate interpretation of R is as the inverse of the intertemporal elasticity of substitution, which in turn is the absolute value of the elasticity of the marginal utility of consumption.

As mentioned in the introduction, we only study relatively far-sighted consumers, who have $\delta < r$. From (3) we know that the consumers will have a non-decreasing consumption path. That is, they will save, and thus build up their wealth and consequently their consumption over time.

However, the latter conclusion disregards σ, the intertemporal elasticity of substitution. Any consumer with a Leontief intertemporal utility function, that is, with $\sigma = 0$, will have a fixed consumption level, independently of their time-preference rate. The permanent consumption level will be $\hat{c} = y + rA_0$.

As a theoretical result, this observation has limited merit. However, we will argue that the observation has important empirical implications, which have not been fully incorporated in our understanding of low consumption traps. Within development economics we have rather focused on factors such as credit constraints, imperfect markets, weak institutions, and coordination problems. The observation may thus have implications for policy and for empirical research. To emphasize the observation it is formulated as Proposition 1.

The proof of the proposition is straightforward, insert $\sigma = 0$ into (3), which implies $g = 0$, and solve for the permanent consumption level $\hat{c}$ using (2).

Proposition 1

Consumers with a Leontief intertemporal utility function, that is with $\sigma = 0$*, will have the permanent consumption level* $\hat{c} = y + rA_0$.

The Leontief intertemporal utility function is illustrated for a low-income consumer (with the permanent utility U_0) in Figure 1, while a high-income consumer (with utility U_t) has $\sigma(c_t) > 0$.

Figure 1 about here

We intentionally illustrate the poor consumer as the one having a Leontief utility function. That is, the figure may illustrate a common per-period utility function $u(c_t)$ where the intertemporal elasticity of substitution $\sigma(c_t) = \frac{-u'(c_t)}{u''(c_t)c_t}$ increases with c_t, and converges towards zero as c_t declines towards a subsistence level *k*. The consumer with $\hat{c} = k$ will have the Leontief utility function. Also note that the wealthy consumer has an increasing consumption path, where $c_t > c_{t-1}$. While Proposition 1 describes the limit case where the poor has a Leontief utility function, we add Assumption 1 to cover the general case, where the preferences converge towards the Leontief function as the permanently sustainable consumption $\hat{c}$ converges towards the subsistence level *k*.

Assumption 1

Assume $\frac{d\sigma(c_t)}{dc_t} > 0$ *for all* $c_t > k$*, and* $\lim_{c_t \to k^+} \sigma(c_t) = 0$.

We interpret *k* as the absolute subsistence level of consumption, that is, we will never observe consumption below *k*. Recall that $\sigma = 1/R$, where *R* is the absolute value of the elasticity of the marginal utility function. That is, the assumption says that the elasticity of the marginal utility function converges towards infinity as consumption converges towards the subsistence level. In this process the absolute value of the second derivative, |*u''*|, converges towards infinity and the marginal utility, *u'*, also increases. So, as *c* declines towards *k*, the utility function becomes steeper, and the change in this slope accelerates. Thus, we may say that the subsistence level of consumption forces the utility function into an extreme curvature that is illustrate in Figure 2, where we have depicted the *u*, *u'* and *u''* functions. In other words, the marginal disutility of savings increases, and at an increasing rate, as consumption converges towards the subsistence level.

The first part of Assumption 1 is parallel to an assumption of decreasing relative risk aversion (DRRA), which is applied within the theory of choice under uncertainty, see for example Deaton (1992). In line with Deaton we note that this is equivalent to $-\frac{u'''(c_t)c_t}{u''(c_t)} > -\frac{u''(c_t)c_t}{u'(c_t)}$, which says that the absolute value of the elasticity of the slope of the marginal utility function is larger than the absolute value of the elasticity of the marginal utility function itself[1].

[1] Note that this is only possible for $u'''(c_t) > 0$, that is, the marginal utility function is convex.

In parallel to $-\frac{u''(c_t)c_t}{u'(c_t)}$, which is the coefficient of relative risk aversion, Deaton (1992) refers to Kimball (1990), who applies $-\frac{u'''(c_t)c_t}{u''(c_t)}$ as a measure of what he denotes as *prudence,* which is the motive for precautionary savings. We thus have another parallel to the theory of choice under uncertainty. However, recall that we do not allow for risk in the present paper, and the expressions will rather be interpreted as the elasticities that we have defined above.

To summarize, we have a characterization of the subsistence level of consumption that is not arbitrary, but rather implicitly defined by the characteristics of the utility function. Furthermore, whenever a consumer is forced to consume close to the subsistence level, he is likely to do so for a long period of time. That is, savings and thus consumption growth, converges towards zero as the permanent income converges towards the subsistence level. This relation is summarized in Proposition 2, which by inspection follows directly from Assumption 1 and equation (3).

Proposition 2

We have $dg/dc_t > 0$ *for all* $c_t > k$*, and* $\lim_{c_t \to k^+} g = 0$.

By Proposition 2 and the budget (2) we get Corollary 1.

Corollary 1

Poor consumers, with a permanently sustainable consumption level $\hat{c}$ *close to (or equal to) the subsistence level k, will have a relatively small (or zero) consumption growth rate.*

Corollary 1 is the main result of the paper. It states that poor households prefer to stay poor for a long period that approaches infinity as $\hat{c}$ converges towards k.

The critical reader may object that Proposition 2, and thus Corollary 1, depends heavily on Assumption 1. This is correct, the theoretical analysis point towards an immediate consequence of a useful definition of a subsistence level of consumption. That is, if consumers have a permanent income near subsistence level, then we shall not expect them to save their way out of poverty in the near future. To illustrate that Assumption 1 is sensible, and that the subsistence constraint is serious, we will, in the next section, apply a reasonably general utility function that has the characteristics specified in Assumption 1, and we use the function to construct a numeric example that illustrates the potential consequences of a subsistence constraint for the savings behavior of the poor.

3. Example

The example we study is the utility function[2]

$$u(c_t) = (ln(c_t / k))^{\alpha} . \qquad (4)$$

[2] Note that this utility function is not equal to $ln(c_t / k)^{\alpha}$, and thus not equal to $\alpha\, ln(c_t / k)$.

As mentioned above, we only observe consumption when $c_t \geq k$, which implies ln(c_t/k) $\geq$ 0. We assume 0 < α < 1, which implies *u' > 0* and *u'' < 0*. To simplify notation, we write (3) as $g = \frac{(r-\delta)}{1/\sigma} = \frac{(r-\delta)}{R}$, and proceed by calculating

$$R = 1 + \frac{1-\alpha}{\ln(c_t / k)}, \tag{5}$$

which implies the characteristics specified in Assumption 1. Inserting for *R* into (3) we have, as an approximation,

$$g_t = \frac{(r-\delta)(c_t - k)}{c_t - \alpha k}, \tag{6}$$

which reduces to *g* = 0 for $c_t = \hat{c} = k$.

The approximation is always true for small c_t*/k* ratios, as *ln*(c_t*/k*) is approximately equal to (c_t - *k)/k*. We will argue that the approximation is also true for large c_t*/k* ratios. A large c_t*/k* ratio means either a small *k*, which we may interpret as the case where the utility function is a profit function, or it means that the permanently sustainable consumption $\hat{c}$ is well above the subsistence level. In both cases we find it reasonable that *R*, and thus the intertemporal elasticity of substitution σ, reduces to 1, and consequently *g* reduces to $(r-\delta)$. By inspection we can see that if c_t is large compared to *k*, then this is also the case for *g* in (6). So, in that case the consumer behaves as if the utility function is linear. To conclude, the specified utility function allows the intertemporal elasticity of substitution σ to increase with c_t from *zero*, at $c_t = \hat{c} = k$, to *one* for large c_t, and in the process the time-dependent consumption growth rate is approximated by (6).

Let us now, as a reference, look into the case where a wealthy consumer actually have $\sigma = 1$, and thus a constant $g = (r-\delta)$. Then, the consumer will have an increasing consumption path, where the intertemporal budget allows consumption in period 1 to equal $c_1 = (1-\frac{g}{r})\hat{c}$, which we may write as $\frac{(\hat{c}-c_1)}{g} = \frac{\hat{c}}{r}$. That is, the present value $\frac{\hat{c}}{r}$ of the permanently sustainable consumption level $\hat{c}$ may be interpreted as the budget constraint for the consumer's trade-off between the level of growth g and the initial cut in consumption, $\hat{c} - c_1$. In short, fast growth requires a low initial consumption level. Also note that if we insert for $g = (r-\delta)$, then the initial consumption is determined by $c_1 = \frac{\delta}{r}\hat{c}$. Furthermore, we have that in the period defined by $\hat{t} = 1 + 1/r$, the consumption c_t will be approximately equal to $\hat{c}$.

Next, we look into the general case, where the growth rate is time-dependent, and below (or equal to) $(r-\delta)$. Then, from (6), we have $\hat{g} = \frac{(r-\delta)(\hat{c}-k)}{\hat{c}-\alpha k}$ as the growth rate in the period where $c_t = \hat{c}$. Note that the expression is smaller than $(r-\delta)$ as long as $k > 0$. Furthermore, since the growth rate, as specified in (6) is increasing in c_t, we know that $\hat{g}$ is an upper bound for g_t for any $c_t < \hat{c}$. So the growth rate is smaller for every period prior to the period where c_t

= $\hat{c}$, and it is possible to show that the period where c_t equals $\hat{c}$ is still close to $\hat{t} = 1 + 1/r$. As a result, the initial consumption c_1 will be larger than in the reference case, that is, c_1 > $(1 - \frac{\hat{g}}{r})\hat{c}$. Thus, when the subsistence constraint matters, then the initial consumption will be relatively higher (as compared to $\hat{c}$) for households that consume close to the subsistence level, than for more wealthy households.

Inserting for $\hat{g}$, from (6), we have $c_1 > \dfrac{\frac{\delta}{r}\hat{c} + k(1 - \alpha - \frac{\delta}{r})}{1 - \frac{\alpha k}{\hat{c}}}$. Note that as k converges towards zero, the expression converges towards $c_1 = \frac{\delta}{r}\hat{c}$, as in the reference case. Let us also note a necessary condition. We will only observe $c_1 = \dfrac{\frac{\delta}{r}\hat{c} + k(1 - \alpha - \frac{\delta}{r})}{1 - \frac{\alpha k}{\hat{c}}}$ $\geq$ k, which implies $\frac{\delta}{r}\hat{c} > \alpha k$.

We apply this necessary condition when we construct the numeric example below.

With initial consumption being larger, and the growth rate being smaller than in the reference case of a constant growth rate, we have two counteracting effects with respect to the time it takes for consumption to reach $\hat{c}$. However, as said, it is possible to demonstrate that, for a small $\hat{g}$, the period where c_t equals $\hat{c}$, will be close to $\hat{t} = 1 + 1/r$.

To conclude, any consumer will start out with $c_1 \geq (1 - \frac{\hat{g}}{r})\hat{c}$, where $\hat{g} = \frac{(r-\delta)(\hat{c}-k)}{\hat{c}-\alpha k}$ is smaller the closer the permanently sustainable consumption level $\hat{c}$ is to the subsistence level *k*. Also note that if $k = 0$, then g is constant and equal to $(r-\delta)$. Furthermore, c_1 tends to be relatively closer to $\hat{c}$ the closer $\hat{c}$ is to *k*. The small relative savings for the poor, during the few periods prior to $\hat{t}$, implies a relatively low growth rate in period $\hat{t}$, as compared to the growth rate for the more wealthy consumers at the same period. Thus, even though the poor might be far-sighted, their consumption growth rate will be low as compared to the rate for more wealthy consumers with the same time-preference rate. We shall now illustrate that the poor may prefer to stay poor even for generations, by way of a numeric example.

So, we complete our argument by applying numeric values to the example developed above. Suppose $k = 2000$ and $\hat{c}$= 2100 for the poor and $\hat{c}$= 3000 for the wealthy, where *c* might be interpreted as the calorie equivalents of the consumer’s daily consumption. We also specify α = 0.5, *r* = 0.3, and δ = 0.25. This interest rate of 30% is representative for long-term informal loans in developing countries. In this numeric example, the poor will start out with a consumption of 2068 (= 0.98$\hat{c}$), which implies an initial growth rate of 0.003, while the wealthy will start out with a consumption of 2750 (= 0.92$\hat{c}$), which implies an initial growth rate of 0.02. In period *four* both types will consume approximately at their $\hat{c}$ level, that is, the poor will consume approximately 2100 and the wealthy approximately 3000. After 30 years the poor will consume 2648, while the wealthy will consume 6182. If the poor rather had a constant rate of growth, and decided to consume 2000 in period 1, then their growth rate would be 0.0143, and their consumption after 30 years would be 3061.

The example illustrates that consumers who differ only in their permanently sustainable consumption level, will not only differ in the level of consumption, but are also likely to differ in growth rates, due to different savings behavior. This is the case even though they pay the same interest rate, have identical time-preference rates and have the same subsistence level of consumption.

4. Conclusions

We apply the theory of optimal intertemporal allocation of consumption to model an endogenous consumption trap. We know from theory that the consumption growth rate is not fully determined by the interest rate and the individual time-preference rate, but also reflects the intertemporal elasticity of substitution, which in turn is determined by the curvature of the per-period utility function. We argue that this insight should influence our understanding of low consumption traps. We believe that the poor stay poor not because of myopic preferences, but rather because of the marginal disutility of savings, which may dominate behavior among households that have permanent income near a subsistence level. Note that there is no production, risk, or credit constraint in the model we apply. Also note that one may use experiments to test our predictions, that is, to investigate whether the time-preference rate and the elasticity of the marginal utility of consumption actually vary with the permanent level of income.

When it comes to real-world savings behavior, we argue that we shall expect the consumption growth rate for poor consumers to be relatively low compared to the growth rate for consumers who only differ from the poor when it comes to their permanent income. That is, the difference in savings can be explained purely by the fact that poor and rich consumers are bound to be at different parts of a common utility function. We thus say that it is reasonable to expect poor households to be relatively less willing to allocate consumption intertemporally, simply because the marginal disutility of savings, as well as the change in this measure, is larger the closer consumption is to a subsistence level.

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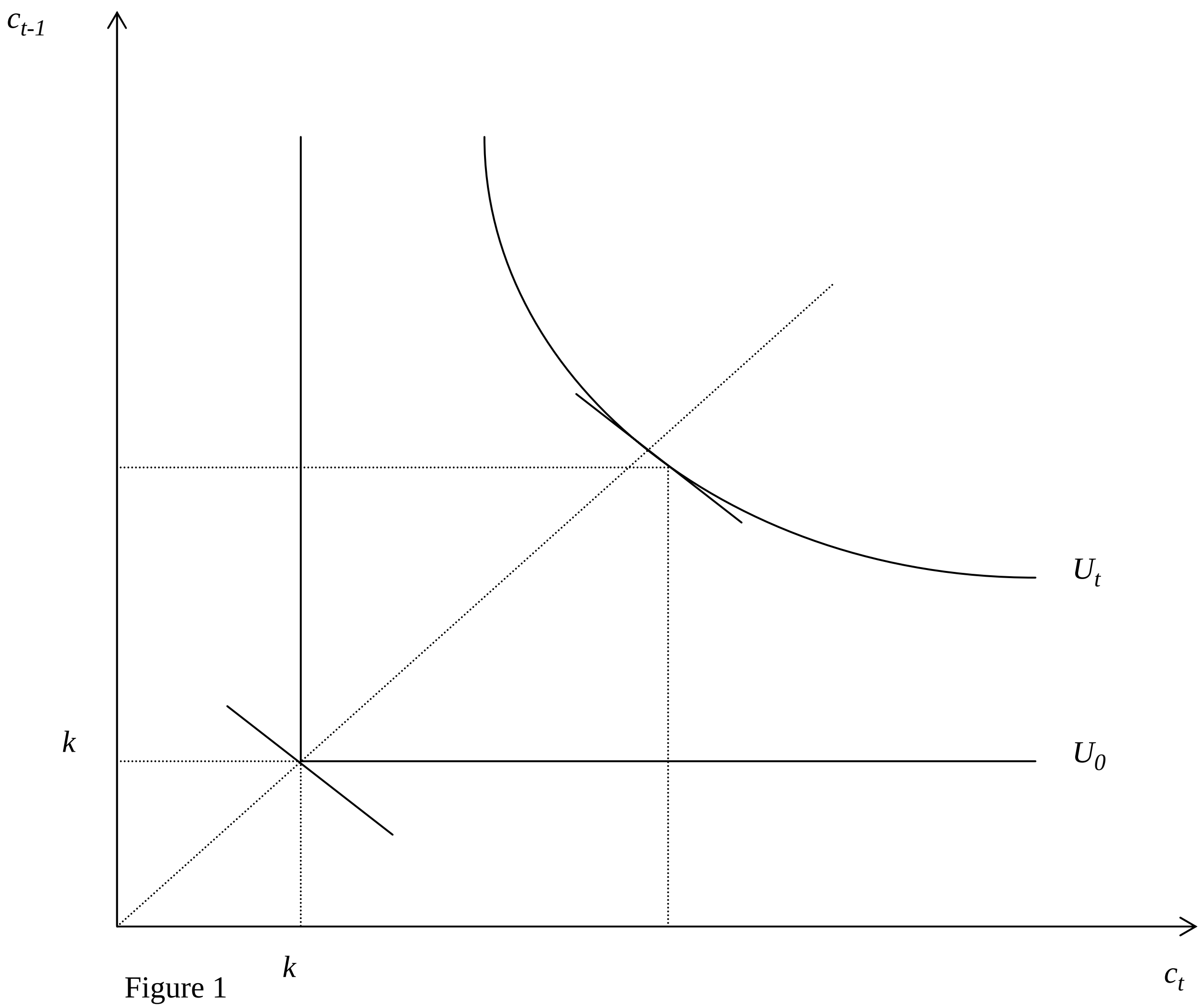

Figure 1

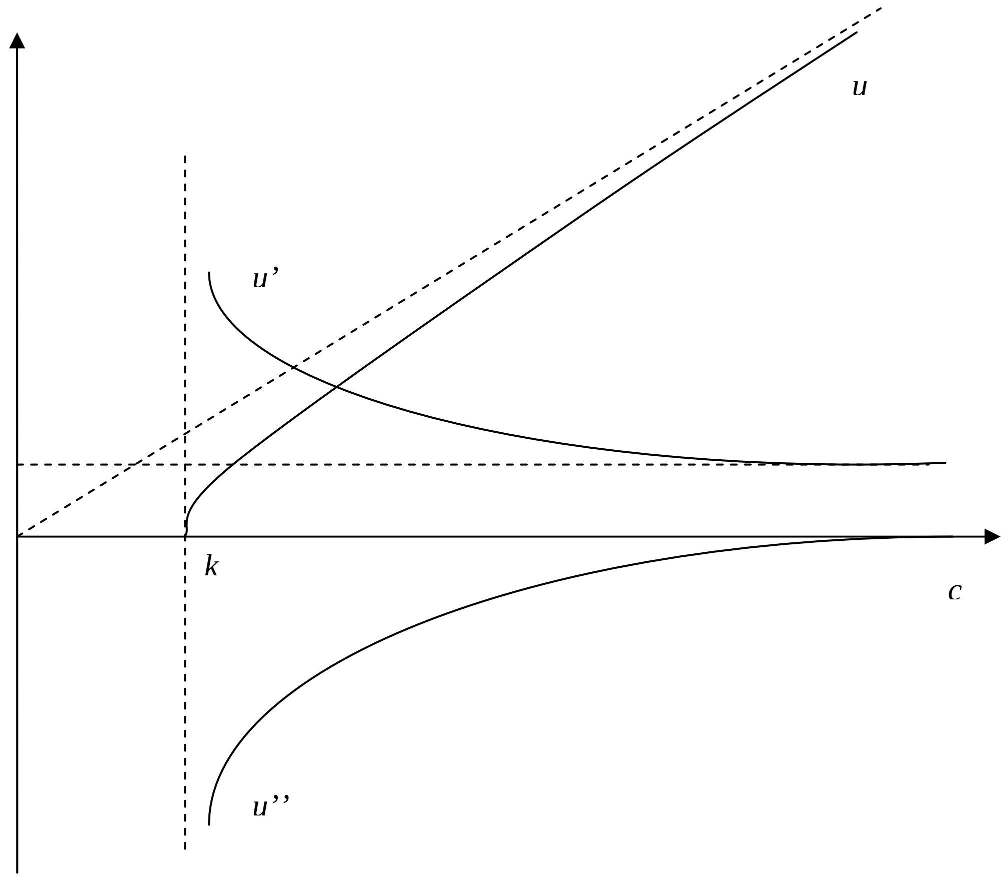

Figure 2