Eliciting People’s Preferences for the Distribution of Health: A Procedure for a more Precise Estimation of Distributional Weights

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1. Introduction

In standard cost-effectiveness analysis, the total benefit of health programmes is measured as the simple sum of individual health gains; the most beneficial programme is the one that produces the highest aggregate health gain. This way of ranking the goodness of alternatives – the *sum-ranking* procedure – is completely insensitive to the distribution of health among individuals; health gains of a given magnitude are counted equally whether they accrue to people with very low lifetime expected health or to people with high lifetime health. This property has lead several authors to call for methods that take into account the equity aspects of the allocation of health benefits (e.g., Broome (1988), Culyer (1989), Wagstaff (1991), Dolan (1998)).

The relevance of the equity aspect is supported by empirical studies, which demonstrate that people’s ethical preferences in general do not conform to the sum-ranking principle (see overview by Dolan et al. (2005)). For instance, a majority of available studies suggest that people tend to give lower priority to old people (e.g., Busschbach et al. (1993), Cropper et al. (1994), Nord et al. (1996), Johannesson and Johansson (1997), Tsuchiya et al. (2003)). Moreover, people tend to put higher weight on health improvements for the more severely ill (Nord (1993, 1999), Ubel (1999), Nord et al. (1999), Brock (2002)). The tendency to assign higher weight to the least well off is confirmed in studies using QALYs as the measuring unit (Bleichrodt et al. (2004, 2005)).

One way of taking care of distributional concerns in cost-effectiveness analysis is by incorporating distributional weights (Williams (1997)). The idea is to assign higher weights to the health gains and costs of individuals who deserve especially favourable treatment for distributional reasons – who, typically, will be the worse off individuals. By using the weighted sum of benefits and costs over all individuals as the measure of goodness, the ranking of alternatives will then incorporate distributional concerns. Thus, the inclusion of distributional weights does not necessarily imply that we abandon the sum-ranking
principle; we may simply replace the unweighted sum ranking by a weighted sum-ranking procedure.¹

In the health economics literature, distributional weights are usually thought to be partly – if not exclusively – a function of people’s health. Since health programmes may have large impacts on the health status of people, distributional weights cannot take the form of a single exogenous parameter per individual. Rather, distributional weights must be specified as a *distributional weighting function* which ascribes a weight to each particular level of health. The valuation of individual health gains will then depend both on the weight attached to the initial level of health as well as on the weights attached to each level of health between the initial and the expected final level of health.

The aim of this paper is to outline a methodology for estimating points along such a distributional weighting function, or more precisely, a methodology that will determine the marginal value that people put on health gains at a given initial level of health. The paper is motivated by our unsuccessful attempt to derive an age weighting function from the existing empirical literature on age weights. But although our motivating examples will be drawn from the age weighting literature, the fundamental problem that we are addressing is equally relevant for other types of health weights (e.g., severity weights or QALY weights).

Most of the empirical literature on age weights is based on the valuation of non-marginal health gains. Two main methodological approaches have been used to elicit people’s age weights: 1) the *person trade off* (PTO), where a health gain of one person at a given age is valued against an identical health gain for *n* persons at a different age (e.g., Cropper et al. (1994), Johannesson and Johansson (1996, 1997), Nord et al. (1996)), and 2) the *gain trade off* (GTO),² where the health gain of one person at a given age is valued against a health gain of a different magnitude to one person at a different age (e.g., Busschbach et al. (1993), Rodriguez and Pinto (2000)).

¹ There are ways of incorporating distributional concerns that do abandon the sum-ranking procedure. Wagstaff (1991) approaches the question of distribution in health by postulating a general class of social welfare functions which is more general than the class of additively separable functions (which is the class that implies sum ranking of alternatives). One non-sum-ranking procedure that has been used in practice is the Cobb-Douglas social welfare function (Dolan, 1998).

² Thanks to Trygve Ottersen for suggesting this terminology.
In principle, the PTO is the method that can most easily be used to study the valuation of marginal health gains. Respondents may simply be asked to value the one extra life year for a person of age $x$ versus one extra life year for $n$ persons of age $y$. However, none of the empirical contributions have followed this approach. Cropper et al. (1994) and Johannesson and Johansson (1996, 1997) have investigated the relative value of saving lives at different ages. But this information is of little help in estimating the weight at age $x$ relative to the weight at age $y$. To say that saving the life of a 20 year old is $n$ times as valuable as saving the life of a 60 year old gives information about the relative average age weights over the remaining lifetime for the two age groups; little or nothing can be inferred about the age weight at age 20 relative to age 60 from this information.

Nord et al. (1996) come somewhat closer to estimating the value of marginal health gains in their analysis of the relative value of extending life at different ages. However, they use a rather long life extension of 10 years as their baseline. To say that to extend the life of a 60-year-old by 10 years is equally valuable as extending the life of a 20-year-old by $n$ years provides information about the average age weight for ages between 60 and 70 years relative to the average age weight for ages between 20 and $20+n$ years. But as will be clearly demonstrated below, these relative average weights may differ substantially from the age weight at age 20 relative to age 60.

Even though the PTO may in principle be used to elicit the valuation of marginal health gains, there are other arguments against this method. The PTO method is based on the normative assumption that a large health gain to a single person can be replaced by a very small health gain distributed across a large number of people. Scanlon (1998) has disputed views of this kind, pointing out that aggregation across lives fails to take the separateness of persons seriously. The loss of a large health gain for one person cannot be compensated, in a moral sense, by the aggregated sum of small health gains to other persons. We do not take a stand on this controversy here, but given the possibility that there are problems related to aggregation across individuals, at least in some cases, we think it is useful to investigate the issue of aggregation separately from the question of distributional weights. This is an
argument for using the GTO rather than the PTO in the elicitation of distributional preferences.

Non-marginal health gains are in principle more likely to be a problem with the GTO than with the PTO method, because the GTO is based on the valuation of health gains of different magnitudes. Respondents may of course be presented with only extremely small health gains, but most people would probably not be able to discriminate between alternatives to this degree (Rodriguez and Pinto, 2000). In order to reduce the cognitive demand on the respondents, larger health gains should be invoked, but then the problems related to non-marginality will set in.3

In order to come to grips with this dilemma, this paper suggests a method that can be used to calculate people’s valuation of marginal health gains based on their response to questions about non-marginal health gains and thus produce more reliable point estimates along distributional weighting functions, such as an age weighting function. The method circumvents the pitfalls of the above-mentioned approaches without imposing additional informational requirements.

In Section 2, we demonstrate that to use estimates of age weights based on the valuation of non-marginal life extensions as approximations of the marginal valuation of life extensions may lead to serious mistakes, even when the age weights have been derived by comparing health programmes with moderate life-extending effects. We then propose a procedure by which the problem may be addressed (Section 3). In Section 4, we demonstrate how the proposed procedure performs in practice with different assumptions about the shape of the underlying health weighting function. Section 5 concludes.

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3 Rodriguez and Pinto (2000) and Busschenbach et al. (1993) both present analysis with two years or less of improved health / extended life. This obviously comes quite close to marginal analysis, but one can question whether the health gains are too small to obtain precise statements about their relative marginal value. The lack of precision is particularly evident in Busschenbach et al., where the methodology based on continuous halving of the time of illness of one of the patients – combined with using the midpoint between the preference switching points as the estimate of the true preference – in effect implies that all age weights between 1 and 2 are approximated by 1.4 (≈2^{0.5}), and all age weights in the interval between 2 and 4 are approximated by 2.8 (≈2^{1.5}), etc.
2. The problem

We will first demonstrate that to use people’s valuation of non-marginal health gains as an approximation of the underlying distributional weights may lead to strongly biased estimates of the weighting function. Consider the problem of eliciting people’s age weights. Assume that people’s true age weights can be represented by the bold curve in Figure 1. This age weighting function represents the weight that people attach to each consecutive life year. Thus, the value of extending the life of a person of age $y_i$ by $\varepsilon_i$ years is equal to the area $A$, i.e., the integral under the weighting function on the interval $(y_i, y_i + \varepsilon_i)$, and the expected value of saving the life of a person of age $y_i$ is equal to the integral under the weighting function from age $y_i$ until the expected age of natural death.

Suppose that we want to know the weight that people assign to age $y_i$ relative to age $y_R$. If the age weighting function were observable, we would immediately see that these ages have the same weight (see Figure 1). In practice, the age weighting function is unobservable, and in order to elicit the age weights, we will attempt to use a standard gain trade-off procedure, asking respondents to state how many extra life years $\varepsilon_i$ to a person aged $y_i$ years would be equivalent to $\varepsilon_R$ extra life years to a person aged $y_R$ years.

In Figure 1, the value of $\varepsilon_R$ extra life years to a person aged $y_R$ years is equal to the area $B$. The respondent then chooses $\varepsilon_i$ so that the area $A$, i.e., the area under the age weighting function on the interval $(y_i, y_i + \varepsilon_i)$, is equal to the area $B$. In this example, $\varepsilon_i$ will clearly be smaller than $\varepsilon_R$. From this fact it may be tempting to conclude that respondents assign a higher age weight to age $y_j$ than to age $y_R$, which clearly is not true; both ages have the same age weight.
It should come as no surprise that a methodology that is based on questions about non-marginal life extensions will not in general be able to provide precise estimates of age weights at the margin. The question is whether the problem is big enough to worry about. The answer seems to be yes: age weights that are derived from questions about non-marginal life extensions (henceforth called the observed age weights) may seriously misrepresent people’s actual distributional preferences (henceforth denoted as the true age weights). We will show this by constructing two examples where the observed age weights are compared with the true ones.

Assume that the true age weighting function takes either of the following forms;

(a) **Linear:** \( w(y) = 8 - 0.1y \)

(b) **Hump-shaped:** \( w(y) = 0.44 ye^{-0.0007y^2} \)

Example (a) assumes that the true age weighting function is linear and that age weights decline monotonically with age, while example (b) assumes a humped-shaped weighting function which assigns the highest weight to ages around 30 years. The parameters of both functions have been chosen in order to reflect the empirical results obtained by Nord et al.
(1996) and Johannesson and Johansson (1996), suggesting that the weights for life years gained at ages between 20 and 30 may be as much as 10 times higher than the weights for life years gained at ages around 70-80. (The functions are, however, in no other respect meant to represent the results from these studies.)

Figures 2a and 2b illustrate the assumed true age weighting functions that are postulated above. In addition, the figures depict the observed age weighting functions, i.e., the weights that would be calculated by using the standard procedure described above, while assuming that people’s answers accurately reflect their true underlying age weighting function. In both cases, we have assumed that people have been asked to state the number of extra life years for a person with a life expectancy of $\nu_i$ years that would be equivalent to 10 extra life years for a person with a life expectancy of 70 years. The large difference between the curves in both examples shows that a standard method used to measure age weights may provide very inaccurate estimates of the true age weights at the margin.

**Linear weighting function**

![Figure 2a: Elicitation of distributional weights from valuation of non-marginal health gains](image-url)
Hump shaped weighting function

The source of the discrepancies between the observed and the true weights is that the observed weights are inferred from non-marginal health improvements; they represent – at each age level $y$ – the *average* age weights for life years between $y$ and $y + \varepsilon$, relative to the *average* age weights for life years between 70 and 80. Even though the value of $\varepsilon$ may be relatively small in our examples (e.g., in the linear example, the value of $\varepsilon$ is less than one year for all ages less than 30 years), the discrepancies are substantial. The problem, of course, is principally the same irrespective of which kind of weights (age weights, severity weights or QALY weights) that we are estimating.

### 3. Towards a higher level of precision

How can this problem be dealt with in practical empirical work? Given that we do not want to mix the potential aggregation problems associated with the PTO method with the elicitation of distributional weights, and given that it is too demanding to ask people directly about their valuation of very small health gains, we need to find ways of utilising the information obtained from non-marginal valuations in a more efficient way. Moreover, since a considerable amount of data has already been produced based on people’s valuation of non-marginal health improvements, it would be interesting to estimate more accurate distributional weights from these data, without having to collect additional information. Therefore, we propose an approximation procedure by which available information on the
valuation of non-marginal health improvements can be utilised more efficiently in order to come closer to the true estimates of age weights at the margin.

Consider a society with \( n \) individuals. Let \( y_j \) denote the expected lifetime health of individual \( j \). It is not essential for our argument how \( y_j \) is measured; one possibility would be to interpret \( y_j \) as the expected number of life years, but it can for instance also represent the expected number of QALYs.

The allocation of health among individuals can be expressed by a health profile \((y_1, \ldots, y_n)\). We assume that the social planner’s preferences over health profiles can be expressed by a social welfare function \( W = W(y_1, \ldots, y_n) \), and that the welfare function can be expressed as a weighted sum of each individual’s health,

\[
W = \sum_j \bar{w}(y_j)y_j,
\]

where \( \bar{w}(y) \) denotes the average weight (per unit of health) for a person with health status \( y \).\(^4\) \( \bar{w}(y) \) is the average weight assigned to health units between 0 and \( y \). Now, let \( w(y) \) denote the weight assigned to health unit \( y \). \( w(y) \) is the *health weighting function* (corresponding to the age weighting function discussed above). The average weight assigned to health units between 0 and \( y \) can now be expressed as

\[
\bar{w}(y) = \frac{\int_0^y w(x)dx}{y},
\]

\(^4\) This formulation implies that our distributional weights may capture more than pure distributional concerns; there may, for instance, be an element of diminishing marginal utility of health that is also captured by these weights. Bleichrodt et al. (2004, 2005) have shown how these aspects can be separated if one is willing to assume that distributional weights are rank ordered (and thus do not depend on the absolute level of health). We find this assumption to be a rather strong one. Moreover, according to Bleichrodt et al. (2005), diminishing marginal utility of health seems to play a negligible role in practice. We therefore stick to an approach where we have to accept that the distributional weights may carry more information than strictly distributional concerns.
implying that the social welfare function can be written as

\[ W = \sum_j \int_0^{y_j} w(x)dx. \]

Social welfare is thus the sum over all individuals of the social value of their respective health states, and the social value of the health allocated to individual \( j \) can be expressed as the area under the health weighting function on the interval \([0, y_j]\).

The shape of the weighting function will depend on people’s ethical preferences. If people count all health units equally, \( w(y) \) will simply be a constant \( k \), and the social welfare function will be \( W = k \sum_j y_j \), which represents the unweighted sum-ranking procedure used in standard cost-effectiveness analysis. A social preference favouring people with low health expectancy (e.g., a low expected age or high severity of disease) would imply a health weighting function that is decreasing in \( y \), with higher weights assigned to the first health units. A humped-shaped weighting function is also conceivable, reflecting, for instance, higher weights on life years in which people typically take on large social responsibilities (e.g., caring for children).

**Eliciting the health weighting function**

The empirical problem is to elicit the health weighting function \( w(y) \) from individual’s preferences over the distribution of health. Since distributional preferences are likely to differ among individuals, we have to distinguish between a) eliciting individuals’ preferences for distribution of health and b) aggregating these preferences into a social weighting function. This paper is only concerned with the first of these issues.

We now present a procedure for eliciting each individual’s health weighting function. We propose to do this through a non-parametric approach where respondents are asked to reveal the number of health units \( \varepsilon \) given to an individual with expected health \( y \) that would be equally valuable as giving \( \varepsilon_R \) extra units of health to a reference individual with
expected health $y_R$. By repeating the question for different levels of $y$, we will be able to extract the information that is needed in order to estimate the weights assigned to various levels of health.

Formally, we pick a vector of health states $y_i$ $(y_1, y_2, ..., y_k)$. It will be useful to fix the interval between the health states to a constant $\mu$ (i.e., $y_{i+1} - y_i = \mu$). The respondent then states a corresponding vector of equivalent health gains $(\varepsilon_1, \varepsilon_2, ..., \varepsilon_k)$, where all $\varepsilon_i$ are chosen in order to satisfy

\begin{equation}
\int_{y_i}^{y_i + \varepsilon_i} \omega(x)dx = \int_{y_R}^{y_R + \varepsilon_R} \omega(x)dx, \quad \forall y_i.
\end{equation}

This equation says that the health gain $\varepsilon_i$ given a person with expected health $y_i$ should be chosen so that the total value of these extra health units equals the total value of $\varepsilon_R$ extra health units to the reference person. In the language of Figure 1, respondents are asked to choose $\varepsilon_i$ for each level of $y_i$ so that area A is the same size as area B.

Areas A and B in Figure 1 provide information about the average weights on the health span $y_i + \varepsilon_i$ relative to the health span $y_R + \varepsilon_R$. In order to come closer to the health weights at $y_i$ and $y_R$, we propose a two-step procedure. In step one, we assume that the true health weighting function can be approximated by a sequence of locally linear functions. By using a Taylor series expansion we then derive the weight at health state $y_i$ as a function of the weight at health state $y_R$ and the slope of the health weighting function at $y_i$ and $y_R$. Furthermore, we reduce the number of unknowns by normalising the weighting function so that $w(y_R)$ is equal to one. In step two, we propose a procedure for approximating the slope of the health weighting function from information about average health weights at various levels of expected health.

For convenience, there is no index on the weighting function, despite the fact that weighting functions are likely to differ across respondents.
Step 1. Linearisation and normalisation. In order to identify \( w(y) \) from the sequence of equivalent health gains \( (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_k) \), we need to impose structure on the form of the health weighting function. For this purpose, we assume that the weighting function can be reasonably approximated by a function that is locally linear.

Definition (LL): The function \( w(y) \) exhibits local linearity if it can be approximated by a linear function on the interval \( (y, y + \varepsilon) \) for small values of \( \varepsilon \).

It seems reasonable to assume that the weighting function is a differentiable function (i.e., a function with no kinks). In this case, the assumption of local linearity is a relatively innocuous one.

If \( w(y) \) is linear on the interval \( (y_i, y_i + \varepsilon_i) \), the area of the trapezium A in Figure 1 can be expressed as

\[
\int_{y_i}^{y_i + \varepsilon_i} w(x)dx = \frac{1}{2} \left[ w(y_i) + w(y_i + \varepsilon_i) \right] \varepsilon_i.
\]

Furthermore, by using a Taylor series expansion and the assumption of local linearity, we can write

\[
w(y_i + \varepsilon_i) = w(y_i) + w'(y_i)\varepsilon_i
\]

Now, by utilising Eqs. (5) and (6), Eq. (4) can be rewritten as

\[
\frac{1}{2} \left[ w(y_i) + w(y_i) + w'(y_i)\varepsilon_i \right] \varepsilon_i = \frac{1}{2} \left[ w(y'_R) + w(y'_R) + w'(y'_R)\varepsilon'_R \right] \varepsilon'_R,
\]

which simply is a restatement of the condition that areas A and B in Figure 1 should be of the same size. The weight at health state \( y_i \) can now be written as
(8) \[ w(y_i) = w(y_{iR}) \frac{e_i}{e_i^2} + \frac{1}{e_i} \left[ w'(y_{iR}) \frac{e_i^2}{2} - w'(y_i) \frac{e_i^2}{2} \right]. \]

The right hand side of Eq. (8) has three unknowns: \( w(y_{iR}) \), \( w'(y_{iR}) \), and \( w'(y_i) \). Normalisation of the weighting function will remove one of them. In practice, the absolute value of the marginal weighting function is of little interest; our interest lies in the relative weights assigned to various health states. Since the social welfare function (3) is unique up to a positive transformation, we can freely choose the scale of the weighting function. We therefore define the normalised marginal weighting function \( w^N \) as

(9) \[ w^N(y) = \frac{w(y)}{w(y_{iR})}. \]

The normalised weighting function is scaled to assign a weight of one to the reference health state (i.e., \( w^N(y_{iR}) = 1 \)). The weights of all health states can now be interpreted as weights relative to the weight of the reference health state.

From Eq. (8), we can now derive an expression for the normalised health weight at health state \( y_i \):

(10) \[ w^N(y_i) = \frac{e_i}{e_i^2} + \frac{1}{e_i} \left[ w^{N'}(y_{iR}) \frac{e_i^2}{2} - w^{N'}(y_i) \frac{e_i^2}{2} \right]. \]

The normalised health weight \( w^N(y_i) \) can be seen as consisting of two components. The first is the inverse of the number of health units given to the group with expected health \( y_i \) that would be equally as good as giving \( e_{iR} \) extra units of health to the reference group. This component is what is usually interpreted as the weight assigned to health state \( y_i \) in empirical research. But as Eq. (10) clearly demonstrates, this estimate is likely to be biased. The second term in Eq. (10) is the correction that is needed in order to take account of the
fact that respondents have revealed information only about average weights assigned over a
 discrete interval of health states, and not about the weight assigned to health state $y_i$.

Eq. (10) shows that without the correction term, the estimated health weights may be biased
 either upwards or downwards, depending on the slope of the health weighting function at
 $y_i$ and $y_R$. The problem is likely to be particularly severe when the weighting function is
 hump shaped, implying that $w^N(y)$ is positively sloped ($w^{N'} > 0$) at some values of $y_i$ and
 negatively sloped ($w^{N'} < 0$) at others. Note also that the problem does not disappear even if
 the health weighting function is linear, as long as $\epsilon_i$ differs from $\epsilon_R$, i.e., as long as the
 weighting function is not horizontal. To our knowledge, this potential bias has not been
 recognised in the previous literature.

**Step 2. Approximating the slope of the weighting function.** In order to solve Eq. (10), we
 need information about the slope of the health weighting function. Such information is not
 readily available, but we propose to use information about differences in average weights in
 the neighbourhood of $y_i$ in order to approximate the slope of $w^N(y_i)$.

The idea is illustrated in Figure 3. We want to estimate the slope of the normalised health
 weighting function $w^N(y)$ at $y_i$. We know the average weight that the respondent assigns
 to health states in the neighbourhood of $y_i$, specifically to health states in the intervals
 $[y_{i-1}, y_i + \epsilon_{i-1}]$ and $[y_{i+1}, y_i + \epsilon_{i+1}]$. In Figure 3, these average health weights are denoted
 $\kappa_{i-1}$ and $\kappa_{i+1}$. We draw a straight line $\alpha$ between the average weights, calculate the slope
 of $\alpha$ and utilise this value as our estimate of the slope of the normalised weighting function
 at $y_i$.

Formally, let $\kappa_i$ denote the average weight assigned to health states on the interval
 $(y_i, y_i + \epsilon_i)$,
\[ (11) \quad \kappa_j \equiv \frac{1}{\epsilon_j} \int_{y_i}^{y_i + \epsilon_j} W(y) dy. \]

Assuming local linearity (LL), it follows straightforwardly that

\[ (12) \quad \kappa_j = w(y_i + \frac{1}{2}\epsilon_i), \]

i.e., \( \kappa_j \) is the health weight of the health state half way between \( y_i \) and \( y_i + \epsilon_i \). We can now write the slope of the line \( \alpha \) in Figure 3 as

\[ (13) \quad \alpha' = \frac{\kappa_{i+1} - \kappa_{i-1}}{\left( y_{i+1} + \frac{1}{2}\epsilon_{i+1} \right) - \left( y_{i-1} + \frac{1}{2}\epsilon_{i-1} \right)}. \]

Then, we use this expression as an estimate of the slope of the normalised health weighting function at \( y_i \)

\[ (14) \quad w_N(y_i) \approx \frac{\kappa_{i+1} - \kappa_{i-1}}{\left( y_{i+1} + \frac{1}{2}\epsilon_{i+1} \right) - \left( y_{i-1} + \frac{1}{2}\epsilon_{i-1} \right)}. \]

Eq. (14) will hold with equality when the weighting function is linear on the whole interval \( [y_{i-1}, y_{i+1}] \), which may be a strong assumption in particular if the interval between these health states is relatively large. In most other cases, Eq. (14) will only be an approximation, though often a rather good one. As demonstrated in Figure 3, the approximation may be quite accurate even when the underlying weighting function is highly non-linear around \( y_i \).

Further examples of the quality of the approximation are provided in Section 4.
A final remaining problem is to estimate the value of \( \kappa_i \). Note that from the definition of \( \kappa_i \), we can write \( \kappa_i = A_i / \varepsilon_i \), where \( A_i \) represents the area under the health weighting function on the interval \((y_i, y_i + \varepsilon_i)\). By definition, the response vector \((\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_k)\) is constructed so that this area is identical for all \( y_i \) (i.e., \( A_i = A_{i+1} = A \)). Assuming local linearity (LL), the area \( A \) can be written

\[
A \equiv \int_{y_i}^{y_i + \varepsilon_i} w^N(y) dy
\]

(15)  \[
= \frac{w^N(y_i) + w^N(y_i + \varepsilon_i)}{2} \varepsilon_i = \frac{w^N(y_i) + \left(w^N(y_i) + w^N(y_i)\varepsilon_i\right)}{2} \varepsilon_i
= w^N(y_i)\varepsilon_i + \frac{1}{2} w^N(y_i)\varepsilon_i^2
\]

Figure 3: Approximating the slope of the health weighting function at \( y_i \)
By replacing $\kappa_i$ in Eq. (14) with $A/\varepsilon_i$ and by utilising the fact that $w^N(y_R) = 1$, Eqs. (14) and (15) yields the following two equations with two unknowns:

\[
(16) \quad w^N(y_R) = A \left( \frac{1/\varepsilon_{R+1} - 1/\varepsilon_{R-1}}{\left( y_{R+1} + \frac{\varepsilon_{R+1}}{2} \right) - \left( y_{R-1} + \frac{\varepsilon_{R-1}}{2} \right)} \right)
\]

\[
(17) \quad A = \varepsilon_R + \frac{1}{2} w^N(y_R) \varepsilon_R^2
\]

Having used these expressions to solve for $A$ and $w^N(y_R)$, we can use Eq. (14) to derive estimates of all $w^N(y_i)$.

Intuitively, what we have done in step two is to use the fact that the normalised weighting function per definition takes the value one at the reference health state $y_R$, and information about the average health weights in the neighbourhood of $y_R$, to estimate the area under the weighting function on the interval $(y_R, y_R + \varepsilon_R)$. Knowing the size of this area ($A$) enables us to calculate average health weights ($A/\varepsilon_i$) along the whole sequence ($\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_k$). Differences in these average health weights can then be used to calculate the slope of the weighting function, $w^N(y_i)$, for all $y_i$.

In summary, our suggested procedure for estimating health weights at a particular level of health from information about the valuation of non-marginal health entails the following steps:

Make the respondent state a vector of equally valuable health gains ($\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_k$) corresponding to a vector of initial health states ($y_1, y_2, \ldots, y_k$).

Select an arbitrary reference health state $y_R$ and normalise the weight at the reference health state to one (i.e., $w^N(y_R) \equiv 1$).
Calculate the slope of the health weighting function at \( y_R \) (i.e., \( w^N(y_R) \)), and the area under the health weighting function on the interval \( (y_R, y_R + \varepsilon_R) \) (i.e., \( A \)), by solving Eqs. (16) and (17).

Utilise the estimate of \( A \) and calculate the slope of the health weighting function for all values of health \( y_i \) (i.e., \( w^N(y_i) \)), using Eq. (14).

Utilise the estimates of the slope of the health weighting function (i.e., \( w^N(y_i) \)) and calculate the health weights from Eq. (10).

One remaining problem is what to do with the end points of the \( y \)-vector (\( y_1 \) and \( y_k \)). In these cases, information about average weights will not be available on more than one side of the observation since \( \kappa_0 \) and \( \kappa_{k+1} \) do not exist in our data set. Sometimes this problem may be dealt with by collecting data beyond the range within which we are interested in the health weights, but this will not always be possible, as where we are interested in the age weight of a newborn. In this case, we will have to relax our information requirements in order to obtain an estimate. The most crude and simple approach would be to assume linearity of the health weighting function on the entire intervals \( [y_1, y_2] \) and \( [y_{k-1}, y_k] \), which implies that we can estimate the slope of the weighting function at \( y_1 \) by the difference in the average weights \( \kappa_2 \) and \( \kappa_1 \), rather than the difference between \( \kappa_2 \) and \( \kappa_0 \) (and similarly for \( y_k \)). The accuracy of this procedure can be improved by collecting some additional data points in the very close neighbourhood of \( y_1 \) and \( y_k \).

4. Empirical illustrations

In this section, we illustrate how our procedure for estimating the valuation of marginal health gains performs when the underlying health weighting function takes on different functional forms. We show that our procedure may perform quite well even when the underlying health weighting function is highly non-linear.
Figures 4a and 4b are the same examples as we introduced in Section 2. In both cases, our starting point is an assumed true health weighting function, which in practice is unobservable. The observed health weighting function is the one that is derived from the standard procedure of asking respondents about their valuation of non-marginal health improvements, as explained in Section 2. The corrected health weighting function is the one that is obtained by using the information obtained from the non-marginal approach in a more efficient way, as outlined in Section 3.\(^6\)

In the linear case, our procedure is able to exactly reproduce the true health weights (see Figure 4a). This is an expected result because then the assumption of local linearity, which we used to calculate the corrected health weights, is unproblematic, implying that Eq. (14) will hold with equality. But our procedure also performs quite satisfactorily in the case of a highly non-linear, hump-shaped health weighting function (see Figure 4b). Further refinements of our procedure are of course possible in order to achieve even better approximations, but our examples indicate that the expected gains from such refinements in terms of higher precision may indeed be quite small.

We also present a third example in Figure 4c, based on the standard DALY age weighting function:

\[
\text{DALY age weights: } w(y) = \beta ye^{-0.04y} .
\]

\(\beta\) is an arbitrary constant which in our example has been set to 0.235, which implies that \(w(70) = 1\), just as in examples (a) and (b).

We assume that people’s true age weights are represented by the DALY age weights. We then calculate the age weights that will be elicited by the standard procedure (i.e., the observed age weights) when respondents are asked to state the number of extra life years to

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\(^6\) The health weights at the endpoints of the \(y\)-vector have been estimated as suggested at the end of Section 3. In the case of the hump-shaped weighting function, we “collected” two additional data points around the end point \(y=70\) (at \(y=67.5\) and \(y=72.5\)) because it seemed unreasonable from our “raw data” that the weighting function could be linear on the interval [60,70].
a person with a life expectancy of $y_i$ years that would be equivalent to 10 extra life years to a person with a life expectancy of 70 years. As shown in Figure 4c, the observed age weights will differ significantly from the true ones. Finally, we calculate the corrected age weights, which turn out to approximate the true age weights reasonably well in this case as well.

![Linear weighting function](image)

*Figure 4a: Perfect approximation in the linear case*

![Hump shaped weighting function](image)

*Figure 4b: Reasonably good approximation in a highly non-linear case.*
5. Final remarks

A distributional weighting function reflects, at each level of health, the valuation of marginal health gains. Knowing the shape of the distributional weighting function would be very useful in cost-effectiveness analysis, because it would facilitate the incorporation of distributional concerns into the economic evaluation of health programmes. However, since most of the empirical literature that has elicited distributional weight for health is based on valuation of non-marginal health gains, it is not straightforward to construct distributional weighting functions from the existing literature.

This paper has demonstrated that a failure to distinguish appropriately between the valuation of marginal and non-marginal health gains may lead to seriously incorrect estimates of the distributional weighting function. The paper has also outlined a methodology that can be employed to utilise the information from non-marginal analysis more efficiently. Our methodology does not, however, produce more than point estimates along the weighting function. In the next step, regression analysis may be applied in order to parameterise a complete weighting function.
References


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SUMMARY

Several empirical studies have demonstrated that people do not evaluate health programmes solely based on aggregate health gains; they also care about the distribution of health. In order to incorporate distributional concerns into cost-effectiveness analysis, it would be useful to elicit distributional weights that express people’s valuation of marginal health gains at various levels of health. Distributional preferences are commonly elicited either through a person trade off (PTO) or a gain trade off (GTO) technique. An inherent problem of the GTO is that it is based on the valuation of non-marginal health gains. In practice, many contributions using the PTO also focus on non-marginal health gains. This paper demonstrates that the failure to distinguish appropriately between marginal and non-marginal health gains may lead to seriously misleading estimates of distributional weights. Moreover, the paper proposes a methodology for utilising information obtained through non-marginal analysis more efficiently in order to obtain more reliable estimates of distributional weights.

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